HEWLETT-PACKARD

Math Pac

Series 80

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NOTICE

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Introduction

The programs in the Math Pac have been drawn from the fields of calculus, numerical analysis, linear systems, geometry, and special functions.

Each program in this Pac is described by a section in this manual. The manual provides a description of the program with relevant equations, a set of user instructions for using the program, and one or more example problems. Program listings can be obtained by loading the desired program and then listing it. The appendix at the back of this manual provides instructions for obtaining the explanatory comments to a particular program which are stored in the comment programs.

If you have already worked through a few programs in the Standard Pac, you will understand how to load a program and how to interpret the User Instructions. If these procedures are not clear to you, take a few minutes to review the sections ''Running a Program'' and ''Format of User Instructions'', in your Standard Pac.

As stated in the Standard Pac, you should define the output peripherals to your needs. Most of the programs assume that the printer is 2 and the CRT is 1 and use PRINT and DISP statements accordingly. If you want to ensure that the peripherals are defined as the programs assume, press RESET before running a program. The currently defined key labels are obtainable at any time while a program is running by pressing KEY LABEL. Remember to press CLEAR -LINE before pressing END LINE if the key labels are in the input line.

We hope that the Math Pac will assist you in the solution of numerous problems in your discipline.

Contents

Program Operation Hints

These programs have been designed to execute with a minimum amount of difficulty, but problems may occur which you can easily solve during program operation. There are four different types of errors or warnings that can occur while executing a program; input errors, math errors, tape errors and image format string errors.

The input errors include errors 43, 44, and 45. These errors will cause a message to be output followed by a new question mark as a prompt for the input. You should verify your mistake and then enter the correct input. The program will not proceed until the input is acceptable. There is a complete discussion of INPUT in your Owner's Manual if you need more detail.

The second type of error which might occur is a math error (1 thru 13). With DEFAULT ON, the first eight errors listed in Appendix E of your Owner's Manual cause a warning message to be output, but program execution will not be halted. The cause of these errors can usually be attributed to specific characteristics of your data and the type of calculations being performed. In most cases, there is no cause for alarm, but you should direct your attention to a possible problem. An example of such a case is found in the Standard Pac when the curve fitting program computes a curve fit to your data which has a value of 1 for the coefficient of determination, r^2. The computation of the F ratio results in a divide by zero, Warning 8.

The third type of error, tape errors (60 thru 75) may be due to several different problems. Some of the most likely causes are the tape being write-protected, the wrong cartridge (or no cartridge) being inserted, a bad tape cartridge, or wrong data file name specification during program execution. Appendix E of your Owner's Manual should be consulted for a complete listing.

The fourth type of error is due to generalizing the output to anticipated data ranges. In many cases, the output has assumed ranges which may or may not be appropriate with your data. Adjusting the image format string for your data will solve this type of problem. You may also want to change the image string if you require more digits to the right of the decimal point.

These are the more common problems which may occur during program operation. Your Owner's Manual should be consulted if you need more assistance.

Notes

Simultaneous Equations

This program solves the system of equations AX = B using Gaussian elimination. It is assumed that the matrix A is square. The triangular matrices overwrite A and the solution matrix X overwrites B. This saves a significant amount of memory, but in the process, the values contained in matrices A and B are destroyed.

The program is designed to handle up to (and including) 25 equations. Once the coefficients have been entered an editing option is available before the equations are solved.

Upon entry into the program, there is a bad data check. If the program detects "nonsense" data, the following error message is printed and the program pauses:

ERROR IN SUBROUTINE
N= (#rows) M= (#columns)

The data may be corrected from the keyboard (e.g., N=5 (END LINE)). When (CONT) is pressed, the program will resume execution at the next line.

If the program detects a row of zeros, the message MATRIX WITH ZERO ROW is printed and the program will pause. In the Gaussian elimination section, there are two tests performed to check that A is not a machine singular matrix. If either test fails, the message MATRIX IS MACHINE SINGULAR is printed.

User Instructions

1. Insert the Math Pac cartridge into the tape transport.
2. To load the program:
   a. Press: (REW LOAD) "SIMEQ"
   b. Press: (END LINE)
3. To start the program:
   a. Press: (RUN)
4. When DIMENSIONS OF B (ROW, COLUMN)? is displayed:
   a. Enter: The dimensions of matrix B, e.g., if B has dimensions 3 by 4, then enter 3,4.
   b. Press: (END LINE)
5. When MATRIX A: is printed and A(I,J)? is displayed:

REFERENCES:

1. Forsythe, G. and Moler, C., *Computer Solution of Linear Algebraic Systems,* (Englewood Cliffs, N.J.: Prentice-Hall, Inc.), Ch. 9, 11.

a. Enter: The I^{th}, J^{th} element of matrix A.
b. Press: END LINE
c. Repeat step 5 until all elements of A are entered.

6. When CHANGES (Y/N)? is displayed:
   a. Enter: Y, if changes are desired in matrix A.
   b. Press: END LINE
   c. Go to step 7.

   OR:

   a. Enter: N, if no changes are desired.
   b. Press: END LINE
   c. Go to step 9.
7. When COORDINATES OF A (ROW,COLUMN)? is displayed:
   a. Enter: The row and column numbers of the element to be changed, e.g., if element A(3, 4) is to be changed, enter 3,4.
   b. Press: END LINE
8. When A(I,J)? is displayed:
   a. Enter: The new value of the element.
   b. Press: END LINE
   c. Go to step 6.
9. When MATRIX B: is printed and B(I,J)? is displayed:
   a. Enter: The I^{th},J^{th} element of matrix B.
   b. Press: END LINE
   c. Repeat step 9 until all elements of B are entered.
10. When CHANGES (Y/N)? is displayed:
    a. Enter: Y, if changes are desired in matrix B.
    b. Press: END LINE
    c. Go to step 11.

    OR:

    a. Enter: N, if no changes are desired.
    b. Press: END LINE
    c. Go to step 13.
11. When COORDINATES OF B (ROW,COLUMN)? is displayed:
    a. Enter: The row and column numbers of the element to be changed, e.g., if element B(3,4) is to be changed, enter 3,4.
    b. Press: END LINE
12. When B(I,J)? (where I and J represent the row and column of the element to be changed) is displayed:
    a. Enter: The new value of the element.
    b. Press: END LINE
    c. Go to step 10.
13. The program will then print the solution matrix X. For a new case, press RUN and go to step 4.

Example 1:

Dimensions of B = 4 × 2

$$\text{Matrix A} = \begin{vmatrix} 1 & -2 & 3 & 1 \\ -2 & 1 & -2 & -1 \\ 3 & -2 & 1 & 5 \\ 1 & -1 & 5 & 3 \end{vmatrix}$$

$$\text{Matrix B} = \begin{vmatrix} 3 & 1 \\ -4 & 0 \\ 7 & 0 \\ 8 & 0 \end{vmatrix}$$

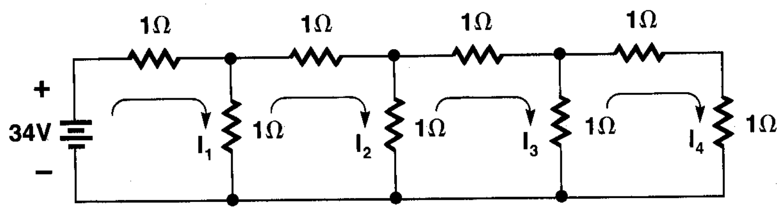
Find Matrix X such that AX = B.

Results:

```
MATRIX X:

X( 1, 1)= 1.000000E+000
X( 1, 2)=-2.884615E-001

X( 2, 1)= 1.000000E+000
X( 2, 2)=-7.307692E-001

X( 3, 1)= 1.000000E+000
X( 3, 2)=-1.923077E-002

X( 4, 1)= 1.000000E+000
X( 4, 2)=-1.153846E-001
```

Example 2:

By applying the technique of loop currents to the circuit below, find the currents I_1, I_2, I_3, and I_4.



The equations to be solved are:

$$\begin{array}{lllll} 2I_1 & -I_2 & & & = 34 \\ -I_1 & +3I_2 & -I_3 & & = 0 \\ & -I_2 & +3I_3 & -I_4 & = 0 \\ & & -I_3 & +3I_4 & = 0 \end{array}$$

Results:

```
MATRIX X:
X( 1, 1)= 2.100000E+001
X( 2, 1)= 8.000000E+000
X( 3, 1)= 3.000000E+000
X( 4, 1)= 1.000000E+000
```

Solution to F(X) = 0 on an Interval

Given a first guess, this program will search for a real root of the equation $F(X) = 0$, where the user defines the continuous real-valued function $F(X)$ starting at line 5000.

Roots are found by marching along at a given step size until a change of sign is encountered. A modified secant method is then used to determine the zero of the function.

The user is required to specify the error tolerances for the root and for the function evaluation, as well as the step size and the maximum number of steps and iterations allowed.

User Instructions

1. Insert the Math Pac cartridge into the tape transport.
2. To load the program:
   a. Type: (REW LOAD) "ROOTS"
   b. Press: (END LINE)
3. Type the function, FNF, to be analyzed starting at line 5000. Make sure that any lines after line 5000 which are not in your function are deleted before running the program.
4. To run the program:
   a. Press: (RUN)
5. When IS FUNCTION STORED (Y/N)? is displayed:
   a. Enter: Y, if the function is stored at line 5000.
   b. Press: (END LINE)
   c. Go to step 6.

   OR:

   a. Enter: N, if the function is not stored.
   b. Press: (END LINE)
   c. Go to step 3.
6. When FIRST GUESS? is displayed:
   a. Enter: A first guess.
   b. Press: (END LINE)
7. When STEP SIZE? is displayed:
   a. Enter: The step size for marching when looking for an interval in which the function changes sign; this must be positive.
   b. Press: (END LINE)
8. When MAXIMUM NUMBER OF STEPS? is displayed:
   a. Enter: The maximum number of allowed steps in marching to find an interval in which the function changes sign.
   b. Press: (END LINE)
9. When MAXIMUM NUMBER OF ITERATIONS? is displayed:
   a. Enter: The maximum number of iterations allowed after an interval has been found in which the function changes sign.
   b. Press: (END LINE)
10. When TOLERANCE FOR ROOT? is displayed:
    a. Enter: The tolerance for the root, i.e., if X_1 and X_2 are two consecutive approximations for the root, the average of X_1 and X_2 is accepted as a root if $|X_1 - X_2| \leq (MAX(1,|X_1|,|X_2|)) * $ (Tolerance for

root)

b. Press: END LINE

11. When TOLERANCE FOR FUNCTION? is displayed:

a. Enter: The tolerance for the function, i.e., an approximation X is accepted as a root if $|F(X)| \leq$ (Tolerance for the function).

b. Press: END LINE

12. The root X as well as the functional value F(X) will be printed. For a new case, press RUN and return to step 5.

Example 1:

Find the root of $\ln x + 3x - 10.8074 = 0$. The function should be stored at line 5000 as shown:

```
5000 DEF FNF(X)=LOG(X)+3*X-10.8074
```

Use 1 as a first guess, .2 as the step size, 20 for the maximum number of steps and iterations and 1E-6 as the tolerance for the root and the function.

```
FIRST GUESS= 1
STEP SIZE= .2
MAXIMUM # OF STEPS= 20
MAXIMUM # OF ITERATIONS= 20
TOLERANCE FOR ROOT= .000001
TOLERANCE FOR FUNCTION= .000001


ROOT= 3.213361E+000
FUNCTION VALUE= 2.300000E-008
```

Example 2:

Find an angle α between 100 and 101 radians such that $\sin\alpha = 0.01$, using a maximum of 20 steps and iterations, a step size of .1, and an error tolerance for the root and the function of 1E-6.

```
5000 DEF FNF(X)=SIN(X)-.01
```

```
FIRST GUESS= 100
STEP SIZE= .1
MAXIMUM # OF STEPS= 20
MAXIMUM # OF ITERATIONS= 20
TOLERANCE FOR ROOT= .000001
TOLERANCE FOR FUNCTION= .000001


ROOT= 1.005410E+002
FUNCTION VALUE=-1.547430E-009
```

Notes

Integration With Equally Spaced Data Points

This program approximates $\int_a^b f(x)dx$ where $f(x)$ is represented by discrete function values $f(x_i)$ at equally spaced points x_i.

The user is required to supply the increment between intervals, the number of data points, which must be an odd integer less than or equal to 999, and the function's value at each of the data points.

Simpson's one-third rule is used to approximate the integral.

$$\int_a^b f(x)dx \approx \frac{h}{3}\{f(a) + 4f(a+h) + 2f(a+2h) + 4f(a+3h) + \ldots + 4f(a+(n-1)h) + f(a+nh)\}$$

where: n = number of intervals (number of data points minus one)

$$h = \frac{b-a}{n}$$

Note: When integrating a function $f(x)$ use Calculus and Roots of $f(x)$ in the Standard Pac.

User Instructions

1. Insert the Math Pac cartridge into the tape transport.
2. To load the program:
   a. Type: (REW LOAD) "INTEG"
   b. Press: (END LINE)
3. To start the program:
   a. Press: (RUN)
4. When # OF DATA POINTS (ODD#)? is displayed:
   a. Enter: The number of data points. (Since Simpson's rule is employed, there must be an *odd* number of points.)
   b. Press: (END LINE)
5. When INCREMENT? is displayed:
   a. Enter: The increment between data points.
   b. Press: (END LINE)
6. When F(I)? (for I = 1 to N) is displayed:
   a. Enter: The appropriate function value of the data point.
   b. Press: (END LINE)
   c. Repeat step 6 until all points are entered.
7. When CHANGES (Y/N)? is displayed:
   a. Enter: Y, if changes are desired.
   b. Press: (END LINE)

   OR:

   a. Enter: N, if no changes are necessary.

REFERENCES:

1. Beckett, Royce and Hunt, James, *Numerical Calculations and Algorithms* (New York: McGraw-Hill, 1967), pp. 166-169.

b. Press: END LINE
c. Go to step 10.

8. When DATA POINT NUMBER? is displayed:
   a. Enter: The data point number of the point to be changed.
   b. Press: END LINE

9. When F(I)? is displayed:
   a. Enter: The function value of the data point.
   b. Press: END LINE
   c. Go to step 7.

10. The program will print the value of the integral. For a new case, press RUN and return to step 4.

Example 1:

```
# OF DATA POINTS= 11
INCREMENT= .1

 FUNCTION VALUES:

    F(  1)= 0.000000E+000
    F(  2)= 1.000000E-001
    F(  3)= 2.000000E-001
    F(  4)= 3.000000E-001
    F(  5)= 4.000000E-001
    F(  6)= 5.000000E-001
    F(  7)= 6.000000E-001
    F(  8)= 7.000000E-001
    F(  9)= 8.000000E-001
    F( 10)= 9.000000E-001
    F( 11)= 1.000000E+000


 INTEGRAL= 5.000000E-001
```

Example 2:

```
# OF DATA POINTS= 9
INCREMENT= .25

FUNCTION VALUES:

    F(  1)= 0.000000E+000
    F(  2)= 2.800000E+000
    F(  3)= 3.800000E+000
    F(  4)= 5.200000E+000
    F(  5)= 7.000000E+000
    F(  6)= 9.200000E+000
    F(  7)= 1.210000E+001
    F(  8)= 1.560000E+001
    F(  9)= 2.000000E+001


INTEGRAL= 1.641667E+001
```

Notes

Integration With Unequally Spaced Data Points

This program approximates $\int_a^b f(x)\,d(x)$ where $f(x)$ is represented by discrete function values for unequally spaced domain values x within the interval $[a, b]$. The user is required to input the data points and the tolerance desired. The number of data points must be less than or equal to 300.

The method implemented in this program involves fitting a curve through the data points and integrating that curve. The curve used is the cubic natural spline function which derives its name from a draftsman's mechanical spline. If the spline is considered as a function represented by $s(x)$, the second derivative $s''(x)$ approximates the curvature.

For the curve through data points $(x_1,y_1), (x_2,y_2), \ldots, (x_n,y_n)$ we want $\int_{x_1}^{x_n} (s''(x))^2\,dx$ to be minimized in order to achieve the "smoothest" curve.

The spline function with minimum curvature has cubic polynomials between adjacent data points. Adjacent polynomials are joined continuously with continuous first and second derivatives as well as $s''(x_1) = s''(x_n) = 0$.

The procedure to determine $s(x)$ involves the iterative solution of a set of simultaneous linear equations by the method of successive overrelaxation. The accuracy to which these equations are solved is specified by the user. For a detailed discussion of the algorithm, see reference 2.

$$\int_{x_1}^{x_n} s(x)dx \approx \sum_{i=1}^{n-1} \tfrac{1}{2}(x_{i+1}-x_i)(y_i+y_{i+1}) - \tfrac{1}{24}(x_{i+1}-x_i)^3\left[s''(x_i)+s''(x_{i+1})\right]$$

Note: When integrating a function $f(x)$ use Calculus and Roots of $f(x)$ in the Standard Pac.

User Instructions

1. Insert the Math Pac cartridge into the tape transport.
2. To load the program:
   a. Type: (REW LOAD) "INTUN"
   b. Press: (END LINE)
3. To start the program:
   a. Press: (RUN)
4. When `# OF DATA POINTS?` is displayed:
   a. Enter: The number of data points (≤300) to be used in the computation.
   b. Press: (END LINE)

REFERENCES:

1. Ralston and Wilf, *Mathematical Methods for Digital Computers,* Vol. II (New York: John Wiley and Sons, 1967) pp. 156-158.
2. Greville, T.N.E., Editor, "Proceedings of an Advanced Seminar Conducted by the Mathematical Research Center", U.S. Army, University of Wisconsin, Madison. October 7-9, 1968. *Theory and Application of Spline Function* (New York, London: Academic Press, 1969), pp. 156-167.

5. When ERROR TOLERANCE? is displayed:
   a. Enter: The error tolerance desired for use in solving the system of equations. A generally acceptable value would be 1E-6.
   b. Press: END LINE
6. When X(I)? (For I = 1 to # of data points) is displayed:
   a. Enter: The X-coordinate of the I^{th} data point. These values must be entered in increasing order.
   b. Press: END LINE
7. When Y(I)? (For I = 1 to # of data points) is displayed:
   a. Enter: The Y-coordinate of the I^{th} data point.
   b. Press: END LINE
   c. If more data points are to be entered, go to step 6.
8. When CHANGES (Y/N)? is displayed:
   a. Enter: Y, if changes in the data are desired.
   b. Press: END LINE
   c. Go to step 9.

   OR:

   a. Enter: N, if no changes in the data are necessary.
   b. Press: END LINE
   c. Go to step 12.
9. When DATA POINT #? is displayed:
   a. Enter: The data point number of the point to be changed.
   b. Press: END LINE
10. When X(I)? is displayed:
    a. Enter: The X-coordinate of the I^{th} data point.
    b. Press: END LINE
11. When Y(I)? is displayed:
    a. Enter: The Y-coordinate of the I^{th} data point.
    b. Press: END LINE
    c. Go to step 8.
12. The program will then print the endpoints of the interval of integration and the value of the integral.

Example 1:

```
# OF DATA POINTS= 5
ERROR TOLERANCE= .000001

DATA POINTS: (INCREASING ORDER)

    X(  1)= 0.000000E+000
    Y(  1)= 0.000000E+000
    X(  2)= 1.000000E+000
    Y(  2)= 1.000000E+000
    X(  3)= 2.000000E+000
    Y(  3)= 2.000000E+000
    X(  4)= 3.000000E+000
    Y(  4)= 3.000000E+000
    X(  5)= 4.000000E+000
    Y(  5)= 4.000000E+000

INTEGRAL FROM    0.00 TO    4.00
IS  8.000000E+000
```

Example 2:

```
# OF DATA POINTS= 8
ERROR TOLERANCE= .000001

DATA POINTS: (INCREASING ORDER)

   X(  1)= 0.000000E+000
   Y(  1)= 0.000000E+000
   X(  2)= 2.000000E-001
   Y(  2)= 4.000000E-002
   X(  3)= 6.000000E-001
   Y(  3)= 3.600000E-001
   X(  4)= 1.000000E+000
   Y(  4)= 1.000000E+000
   X(  5)= 1.100000E+000
   Y(  5)= 1.210000E+000
   X(  6)= 1.500000E+000
   Y(  6)= 2.250000E+000
   X(  7)= 1.600000E+000
   Y(  7)= 2.560000E+000
   X(  8)= 2.000000E+000
   Y(  8)= 4.000000E+000


INTEGRAL FROM   0.00 TO    2.00
IS  2.669976E+000
```

Notes

Ordinary Differential Equations

This program is a fourth-order Runge-Kutta procedure for solving systems of first-order ordinary differential equations. The user must supply the dimension of the system of differential equations, ($\leqslant 3$), the beginning and end points of integration, the initial value vector, the maximum number of integration steps, and the user-defined system of equations to be solved.

Runge-Kutta methods attempt to obtain greater accuracy, and at the same time avoid the need for higher derivatives, by evaluating the function F at selected points on each subinterval. For the equation $y' = f(x,y)$, $y(x_0) = y_0$ and step size h, approximations y_n to $y_{(x_0 + nh)}$ for $n = 0, 1, 2, \ldots$ are generated using the recursion formula

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{1}{2}k_1\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{1}{2}k_2\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

This is a single step method. It requires only the value of y at a point $x = x_n$ to find y and y' at $x = x_{n+1}$.

The step size, h, must be chosen with some care. On the one hand, a small h keeps the error due to the Runge-Kutta formula small. On the other hand, the smaller the h, the more integration steps we shall have to perform, and the greater the round off error is likely to be. In addition, the limitation of N in the formula:

$$N = \mathrm{INT}((B\text{-}A)/h + 1) + 1 \qquad N < 450$$

imposes a restraint on the minimum step size h.

Since there may be some cumulative round off error in stepping from end points, A to B, i.e., $X = A + (N - 1)h$, B may not be reached exactly. So, the program stops if $X > B - 10^{-6}$. If X does not satisfy this condition, the program will stop when the maximum number of steps have been performed. The program prints every tenth value of the computed vectors. If a different number of values is required, line 770 may be changed.

REFERENCES:

1. Ralston and Wilf, *Mathematical Methods for Digital Computers,* Vol. 1, New York: John Wiley and Sons, Inc., 1960, pp. 115.
2. Carnahan, Luther, Wilkes, *Applied Numerical Methods,* New York: John Wiley and Sons, Inc., 1969, pp. 363-366.

A higher order differential equation may be replaced by a system of 1st order equations. Assume the equation is of the form:

$$\frac{d^m y}{dx^m} = F\left(X, Y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots, \frac{d^{m-1}y}{dx^{m-1}}\right)$$

with the given initial conditions $y(x_0)$, $\frac{dy}{dx}(x_0), \ldots, \frac{d^{m-1}y(x_0)}{dx^{m-1}}$. We may write the system as follows:

$$\begin{aligned} Y_1' &= F_1(x, y_1, y_2, \ldots, y_m) \\ Y_2' &= F_2(x, y_1, y_2, \ldots, y_m) \\ &\vdots \\ Y_m' &= F_m(x, y_1, y_2, \ldots, y_m) \end{aligned}$$

User Instructions

1. Insert the Math Pac cartridge into the tape transport.
2. To load the program:
   a. Type: (REW LOAD) "DIFEQ"
   b. Press: (END LINE)
3. Define the system of equations to be solved beginning at line 5000. Be sure to add a RETURN statement.
4. To start the program:
   a. Press: (RUN)
5. When IS FUNCTION STORED (Y/N)? is displayed:
   a. Enter: Y, if the function is stored at line 5000.
   b. Press: (END LINE)
   c. Go to step 6.

   OR:

   a. Enter: N, if the function is not stored.
   b. Press: (END LINE)
   c. Go to step 3.
6. When DIMENSION OF SYSTEM OF D.E.? is displayed:
   a. Enter: The dimension of the system of differential equations (≤3).
   b. Press: (END LINE)
7. When LOWER BOUND? is displayed:
   a. Enter: The lower bound of the interval.
   b. Press: (END LINE)
8. When UPPER BOUND? is displayed:
   a. Enter: The upper bound of the interval.
   b. Press: (END LINE)
9. When STEP SIZE? is displayed:
   a. Enter: The step size for integrating.
   b. Press: (END LINE)
10. When MAX # OF INTEGRATION STEPS? is displayed:
    a. Enter: The maximum number of integration steps permitted. This enables the user to put an additional constraint on the number of integrations performed.
    b. Press: (END LINE)
11. When INITIAL VALUES: is printed and Y(I)? (for I = 1 to the number of dimensions of the system) is displayed:
    a. Enter: The I^{th} initial value.
    b. Press: (END LINE)
    c. Repeat step 11 if additional values are required.
12. The program will print the domain values X, as well as the values of the integrated function at X. For a new case, press (RUN) and return to step 5.

Example 1:

Solve $y' - xy^{1/3} = 0$ for $1 \leqslant x \leqslant 3$, step size $= 0.01$, maximum number of integration steps $= 201$, subject to the initial condition $y(1) = 1$. Print every tenth value. Define the system of equations as follows:

```
    5000 F(1) = X*Y(1)^(1/3)
    5010 RETURN


DIMENSION OF SYSTEM OF D.E.= 1
LOWER BOUND= 1
UPPER BOUND= 3
STEP SIZE= .01
MAX # OF INTEGRATION STEPS= 201

INITIAL VALUES:

    Y( 1)= 1.000000E+000

          X                   Y

   1.000000E+000      1.000000E+000
   1.100000E+000      1.106817E+000
   1.200000E+000      1.227880E+000
   1.300000E+000      1.364136E+000
   1.400000E+000      1.516565E+000
   1.500000E+000      1.686171E+000
   1.600000E+000      1.873982E+000
   1.700000E+000      2.081045E+000
   1.800000E+000      2.308421E+000
   1.900000E+000      2.557187E+000
   2.000000E+000      2.828427E+000
   2.100000E+000      3.123239E+000
   2.200000E+000      3.442725E+000
   2.300000E+000      3.787995E+000
   2.400000E+000      4.160166E+000
   2.500000E+000      4.560359E+000
   2.600000E+000      4.989698E+000
   2.700000E+000      5.449312E+000
   2.800000E+000      5.940333E+000
   2.900000E+000      6.463894E+000
   3.000000E+000      7.021132E+000
```

Example 2:

Solve $y'' + 2yy' = 0$ for $0 \leqslant x \leqslant 2$, step size = .01, 201 integration steps, subject to the initial conditions $y(0) = 0$, $y'(0) = 1$. Print every tenth value. The solution may be obtained by first reducing the equation by substitution to a set of simultaneous equations:

$$z = y'$$
$$z' = -2yy'$$

Using the notation $Y(1) = y$ and $Y(2) = y'$, the subroutine is set up as follows:

```
5000 F(1) = Y(2)
5010 F(2) = -2*Y(1)*Y(2)
5020 RETURN

DIMENSION OF SYSTEM OF D.E.= 2
LOWER BOUND= 0
UPPER BOUND= 2
STEP SIZE= .01
MAX # OF INTEGRATION STEPS= 201

INITIAL VALUES:

   Y( 1)= 0.000000E+000
   Y( 2)= 1.000000E+000

        X                 Y

  0.000000E+000     0.000000E+000
  1.000000E-001     9.966799E-002
  2.000000E-001     1.973753E-001
  3.000000E-001     2.913126E-001
  4.000000E-001     3.799490E-001
  5.000000E-001     4.621172E-001
  6.000000E-001     5.370496E-001
  7.000000E-001     6.043678E-001
  8.000000E-001     6.640368E-001
  9.000000E-001     7.162979E-001
  1.000000E+000     7.615942E-001
  1.100000E+000     8.004990E-001
  1.200000E+000     8.336546E-001
  1.300000E+000     8.617232E-001
  1.400000E+000     8.853516E-001
  1.500000E+000     9.051483E-001
  1.600000E+000     9.216686E-001
  1.700000E+000     9.354091E-001
  1.800000E+000     9.468060E-001
  1.900000E+000     9.562375E-001
  2.000000E+000     9.640276E-001
```

Solve $yy'' + 3(y')^2 = 0$ for $0 \leqslant x \leqslant 2$, with step size = .01 and 201 integration steps, subject to the initial conditions $y(0) = 1$, $y'(0) = 1/4$. The following substitutions reduce the equation to a set of simultaneous equations:

$$z = y'$$
$$z = -3(y')^2/y$$

Using the notation $Y(1) = y$ and $Y(2) = y'$, the subroutine is set up as follows:

```
5000 F(1) = Y(2)
5010 F(2) = -3*Y(2)^2/Y(1)
5020 RETURN
```

```
DIMENSION OF SYSTEM OF D.E.= 2
LOWER BOUND= 0
UPPER BOUND= 2
STEP SIZE= .01
MAX # OF INTEGRATION STEPS= 201

INITIAL VALUES:

   Y( 1)= 1.000000E+000
   Y( 2)= 2.500000E-001

         X                    Y

   0.000000E+000     1.000000E+000
   1.000000E-001     1.024114E+000
   2.000000E-001     1.046635E+000
   3.000000E-001     1.067790E+000
   4.000000E-001     1.087757E+000
   5.000000E-001     1.106682E+000
   6.000000E-001     1.124683E+000
   7.000000E-001     1.141858E+000
   8.000000E-001     1.158292E+000
   9.000000E-001     1.174055E+000
   1.000000E+000     1.189207E+000
   1.100000E+000     1.203801E+000
   1.200000E+000     1.217883E+000
   1.300000E+000     1.231493E+000
   1.400000E+000     1.244666E+000
   1.500000E+000     1.257433E+000
   1.600000E+000     1.269823E+000
   1.700000E+000     1 281861E+000
   1.800000E+000     1.293569E+000
   1.900000E+000     1.304967E+000
   2.000000E+000     1.316074E+000
```

Notes

Chebyshev Polynomial

This program fits a tabular function $y(x)$ (given m points (x, y)) to a polynomial

$$P = \sum_{i=0}^{n} A_i X^i$$

This polynomial is the best polynomial approximation of $y(x)$ in the Chebyshev sense. See the two references for a complete description of the method and formula employed.

The polynomial must have degree less than or equal to 10, and the program can handle 450 data points.

User Instructions

1. Insert the Math Pac cartridge into the tape transport.
2. To load the program:
   a. Type: (REW LOAD) "CHEBY"
   b. Press: (END LINE)
3. To start the program:
   a. Press: (RUN)
4. When `# OF DATA POINTS?` is displayed:
   a. Enter: The number of data points to be used in the calculation (# points $\leq$ 450).
   b. Press: (END LINE)
5. When `DEGREE OF POLYNOMIAL?` is displayed:
   a. Enter: The degree of the polynomial desired for the approximation (degree $\leq$ 10). For reasonable results, the degree of the polynomial should be less than the number of data points used minus one, i.e., (degree of polynomial) < (number of data points − 1).
   b. Press: (END LINE)
6. When `DATA POINTS:` is printed and `X(I)?` (for I = 1 to # of data points) is displayed:
   a. Enter: The Ith X-value.
   b. Press: (END LINE)
7. When `Y(I)?` (for I = 1 to # of data points) is displayed:
   a. Enter: The Ith Y-value.
   b. Press: (END LINE)
   c. If more points are to be entered, go to step 6.
8. When `CHANGES (Y/N)?` is displayed:
   a. Enter: `Y`, if changes in the data are desired.
   b. Press: (END LINE)
   c. Go to step 9.

   OR:

   a. Enter: `N`, if no changes are desired in the

REFERENCES:

1. Newhouse, Albert, "Chebyshev Curve Fit", Comm. ACM 5 (May 1962), p. 281.
2. Stiefel, E., *Numerical Methods of Tchebysheff Approximation*, U. of Wisconsin Press (1959), p. 217-232.

data.

b. Press: END LINE

c. Go to step 12.

9. When DATA POINT #? is displayed:

a. Enter: The subscript of the element to be changed.

b. Press: END LINE

10. When X(I)? (where I is the subscript of the element to be changed) is displayed:

a. Enter: The new value of the element.

b. Press: END LINE

11. When Y(I)? (where I is the subscript of the element to be changed) is displayed:

a. Enter: The new value of the element.

b. Press: END LINE

c. Go to step 8.

12. The coefficients of the Chebyshev Polynomial will then be printed where P(X) = A(0) + A(1) * X + A(2) * X ∧ 2 + … + A(N) * X ∧ N

For a new case, press RUN and return to step 4.

Example 1:

```
# OF DATA POINTS= 8

DEGREE OF POLYNOMIAL= 3

DATA POINTS:

   X( 1)=-5.000000E+000
   Y( 1)= 2.500000E+001
   X( 2)= 5.000000E+000
   Y( 2)= 2.500000E+001
   X( 3)=-4.000000E+000
   Y( 3)= 1.600000E+001
   X( 4)= 4.000000E+000
   Y( 4)= 1.600000E+001
   X( 5)=-3.000000E+000
   Y( 5)= 9.000000E+000
   X( 6)= 3.000000E+000
   Y( 6)= 9.000000E+000
   X( 7)=-2.000000E+000
   Y( 7)= 4.000000E+000
   X( 8)= 2.000000E+000
   Y( 8)= 4.000000E+000

COEFF OF CHEBYSHEV POLYNOMIAL:

   A( 0)= 0.000000E+000
   A( 1)= 0.000000E+000
   A( 2)= 1.000000E+000
   A( 3)= 0.000000E+000
```

Example 2:

```
# OF DATA POINTS= 6

DEGREE OF POLYNOMIAL= 3

DATA POINTS:

    X( 1)=-4.000000E+000
    Y( 1)=-6.000000E+001
    X( 2)=-2.000000E+000
    Y( 2)=-8.000000E+000
    X( 3)=-1.000000E+000
    Y( 3)= 1.235600E+000
    X( 4)= 1.000000E+000
    Y( 4)= 1.000000E+000
    X( 5)= 3.000000E+000
    Y( 5)= 2.500000E+001
    X( 6)= 5.000000E+000
    Y( 6)= 1.200000E+002

COEFF OF CHEBYSHEV POLYNOMIAL:

    A( 0)=-3.071429E-001
    A( 1)=-1.357143E-001
    A( 2)= 3.571429E-002
    A( 3)= 9.571429E-001
```

Notes

Fourier Series Coefficients for Equally Spaced Data Points

This program calculates the Fourier series coefficients A(I) and B(I) of the Fourier series corresponding to a function $F(X)$ which is specified by N discrete equally spaced data points $(X(I), Y(I)), I = 1, \ldots,$ N $\leq$ 500.

If: $g(x) = f(x) \cos\left(\frac{2\pi i x}{T}\right)$

and $h(x) = f(x) \sin\left(\frac{2\pi i x}{T}\right)$ then:

$$a_i \approx \frac{2\Delta x}{3T} \{g(x_1) + 4g(x_2) + 2g(x_3) + 4g(x_4) + \ldots + 4g(x_{n-1}) + g(x_n)\}$$

$$b_i \approx \frac{2\Delta x}{3T} \{h(x_1) + 4h(x_2) + 2h(x_3) + 4h(x_4) + \ldots + 4h(x_{n-1}) + h(x_n)\}$$

Sine and cosine functional values are computed recursively with the following formula:

$$\sin\left(\frac{2\pi x_i}{\Delta x}(J+1)\right) = \sin\left(\frac{2\pi x_i}{\Delta x}\right) \cos\left(\frac{2\pi x_i}{\Delta x} J\right) + \cos\left(\frac{2\pi x_i}{\Delta x}\right) \sin\left(\frac{2\pi x_i}{\Delta x} J\right)$$

$$\cos\left(\frac{2\pi x_i}{\Delta x}(J+1)\right) = \cos\left(\frac{2\pi x_i}{\Delta x}\right) \cos\left(\frac{2\pi x_i}{\Delta x} J\right) - \sin\left(\frac{2\pi x_i}{\Delta x}\right) \sin\left(\frac{2\pi x_i}{\Delta x} J\right)$$

For valid results, the user should provide at least N data points to compute N coefficients.

User Instructions

1. Insert the Math Pac cartridge into the tape transport.
2. To load the program:
   a. Type: (REW LOAD) "FOUREQ"
   b. Press: (END LINE)
3. To start the program:
   a. Press: (RUN)
4. When # OF DATA POINTS (ODD#)? is displayed:
   a. Enter: The number of data points to be used in the calculation. There must be an odd number of points.
   b. Press: (END LINE)

REFERENCES:

1. Hamming, R. W., *Numerical Methods for Scientists and Engineers,* McGraw-Hill, 1962, pp. 67-80.
2. Acton, Forman S., *Numerical Methods that Work,* Harper and Row, 1970, pp. 221-257.

5. When HIGHEST COEFFICIENT? is displayed:
   a. Enter: The highest coefficient desired in the Fourier series. This should be less than or equal to the number of data points for valid results.
   b. Press: END LINE
6. When INITIAL DOMAIN VALUE? is displayed:
   a. Enter: The initial domain value, i.e., the first x-value.
   b. Press: END LINE
7. When INCREMENT? is displayed:
   a. Enter: The increment between x-values.
   b. Press: END LINE
8. When RANGE VALUES: is printed and Y(I)? (for I = 1 to N) is displayed:
   a. Enter: The appropriate y-value.
   b. Press: END LINE
   c. Repeat step 8 until all data values are entered.
9. When CHANGES (Y/N)? is displayed:
   a. Enter: Y, if changes are to be made in the data.
   b. Press: END LINE
   c. Go to step 10.

   OR:
   a. Enter: N, if the data is correct.
   b. Press: END LINE
   c. Go to step 12.
10. When DATA POINT #? is displayed:
    a. Enter: The number of the data point to be changed.
    b. Press: END LINE
11. When Y(I)? is displayed:
    a. Enter: The correct y-value.
    b. Press: END LINE
    c. Go to step 9.
12. The program will print the values of the coefficients. For a new case, press RUN and return to step 4.

Example 1:

```
# OF DATA POINTS= 31
HIGHEST COEFFICIENT= 5
INITIAL DOMAIN VALUE= 0
INCREMENT= .1

RANGE VALUES:

    Y(  1)= 1.500000E+000
    Y(  2)= 3.000000E+000
    Y(  3)= 3.000000E+000
    Y(  4)= 3.000000E+000
    Y(  5)= 3.000000E+000
    Y(  6)= 3.000000E+000
    Y(  7)= 3.000000E+000
    Y(  8)= 3.000000E+000
    Y(  9)= 3.000000E+000
    Y( 10)= 3.000000E+000
    Y( 11)= 1.500000E+000
    Y( 12)= 0.000000E+000
    Y( 13)= 0.000000E+000
    Y( 14)= 0.000000E+000
    Y( 15)= 0.000000E+000
    Y( 16)= 0.000000E+000
    Y( 17)= 0.000000E+000
    Y( 18)= 0.000000E+000
    Y( 19)= 0.000000E+000
    Y( 20)= 0.000000E+000
    Y( 21)= 1.000000E+000
    Y( 22)= 2.000000E+000
    Y( 23)= 2.000000E+000
    Y( 24)= 2.000000E+000
    Y( 25)= 2.000000E+000
    Y( 26)= 2.000000E+000
    Y( 27)= 2.000000E+000
    Y( 28)= 2.000000E+000
    Y( 29)= 2.000000E+000
    Y( 30)= 2.000000E+000
    Y( 31)= 1.000000E+000
```

```
COEFFICIENTS:

    A(   0)= 1.638889E+000
    A(   1)= 1.322781E+000
    B(   1)= 4.774700E-001
    A(   2)=-7.448371E-001
    B(   2)= 2.387741E-001
    A(   3)=-5.555556E-002
    B(   3)=-1.286116E-011
    A(   4)= 2.900530E-001
    B(   4)= 1.197223E-001
    A(   5)=-3.333333E-001
    B(   5)= 9.622504E-002
```

Fourier Series Coefficients for Unequally Spaced Data Points

This program calculates the Fourier series coefficients for a function defined by discrete data points $(X(I), Y(I)), I = 1, \ldots, N$. The data pairs must be entered such that the $X(I)$ are discrete, but not necessarily equally-spaced, and $X(I) < X(I + 1)$ for $I = 1, \ldots, N - 1$. $N \leq 300$.

The finite Fourier series is given by the formula:

$$\frac{a_0}{2} + \sum_{i=1}^{N} \left(a_i \cos \frac{2i\pi x}{T} + b_i \sin \frac{2i\pi x}{T} \right)$$

where the Fourier coefficients a_i and b_i are:

$$a_i = \frac{2}{T} \int_{x_1}^{x_1 + T} f(x) \cos \frac{2\pi i x}{T} \, dx \qquad \text{for } i = 0, \ldots, N$$

$$b_i = \frac{2}{T} \int_{x_1}^{x_1 + T} f(x) \sin \frac{2\pi i x}{T} \, dx \qquad \text{for } i = 1, \ldots, N$$

T specifies the period equivalent to $(x_n - x_1)$ and N indicates the number of coefficients desired. The coefficients are evaluated by numerically integrating a parabola passing through three successive points. Execution time depends on the number of coefficients calculated.

For valid results, the user should provide at least N data points to compute N coefficients.

User Instructions

1. Insert the Math Pac cartridge into the tape transport.
2. To load the program:
   a. Type: (REW LOAD) "FOURUN"
   b. Press: (END LINE)
3. To start the program:
   a. Press: (RUN)
4. When # OF DATA POINTS? is displayed:
   a. Enter: The number of data points to be used in the calculation.
   b. Press: (END LINE)

REFERENCES:

Hewlett-Packard 9820A Math Pac, pp. 43-50.

Hamming, R. W., *Numerical Methods for Scientists and Engineers,* McGraw-Hill, 1962, pp. 67-80.

Acton, Forman S., *Numerical Methods that Work,* Harper and Row, 1970, pp. 221-257.

5. When HIGHEST COEFFICIENT? is displayed:
   a. Enter: The highest Fourier series coefficient desired. For valid results, this should be less than or equal to the number of data points provided.
   b. Press: (END LINE)
6. When DATA VALUES: is printed and X(I)? (for I = 1 to N) is displayed:
   a. Enter: The appropriate x-value. The data points should be entered so that the X(I) are discrete and X(I) < X(I + 1) for I = 1 to N. The points need not be equally spaced.
   b. Press: (END LINE)
7. When Y(I)? (For I = 1 to N) is displayed:
   a. Enter: The appropriate y-value.
   b. Press: (END LINE)
   c. Go to step 6 if there are more data values to be entered.
8. When CHANGES (Y/N)? is displayed:
   a. Enter: Y, if changes are to be made in the data.
   b. Press: (END LINE)
   c. Go to step 9.

   OR:

   a. Enter: N, if the data is correct.
   b. Press: (END LINE)
   c. Go to step 12.
9. When DATA POINT #? is displayed:
   a. Enter: The number of the data point to be changed.
   b. Press: (END LINE)
10. When X(I)? (where I is the subscript of the data point to be changed) is displayed:
    a. Enter: The correct x-value.
    b. Press: (END LINE)
11. When Y(I)? (where I is the subscript of the data point to be changed) is displayed:
    a. Enter: The correct y-value.
    b. Press: (END LINE)
    c. Go to step 8.
12. The program will print the resulting A_k and B_k Fourier series coefficients. For a new case, press (RUN) and return to step 4.

Example 1:

```
# OF DATA POINTS= 37
HIGHEST COEFFICIENT= 10

DATA VALUES:

     X(  1)= 0.000000E+000
     Y(  1)= 1.500000E+000
     X(  2)= 1.000000E-001
     Y(  2)= 3.000000E+000
     X(  3)= 1.500000E-001
     Y(  3)= 3.000000E+000
     X(  4)= 2.000000E-001
     Y(  4)= 3.000000E+000
     X(  5)= 2.500000E-001
     Y(  5)= 3.000000E+000
     X(  6)= 3.000000E-001
     Y(  6)= 3.000000E+000
     X(  7)= 4.000000E-001
     Y(  7)= 3.000000E+000
     X(  8)= 5.000000E-001
     Y(  8)= 3.000000E+000
     X(  9)= 6.000000E-001
     Y(  9)= 3.000000E+000
     X( 10)= 7.000000E-001
     Y( 10)= 3.000000E+000
     X( 11)= 8.000000E-001
     Y( 11)= 3.000000E+000
     X( 12)= 9.000000E-001
     Y( 12)= 3.000000E+000
     X( 13)= 1.000000E+000
     Y( 13)= 1.500000E+000
     X( 14)= 1.100000E+000
     Y( 14)= 0.000000E+000
     X( 15)= 1.200000E+000
     Y( 15)= 0.000000E+000
     X( 16)= 1.300000E+000
     Y( 16)= 0.000000E+000
     X( 17)= 1.500000E+000
     Y( 17)= 0.000000E+000
     X( 18)= 1.700000E+000
     Y( 18)= 0.000000E+000
     X( 19)= 1.900000E+000
     Y( 19)= 0.000000E+000
     X( 20)= 2.000000E+000
     Y( 20)= 0.000000E+000
     X( 21)= 2.500000E+000
     Y( 21)= 0.000000E+000
```

```
X( 22)= 2.750000E+000
Y( 22)= 0.000000E+000
X( 23)= 3.000000E+000
Y( 23)= 0.000000E+000
X( 24)= 3.500000E+000
Y( 24)= 0.000000E+000
X( 25)= 3.700000E+000
Y( 25)= 0.000000E+000
X( 26)= 3.900000E+000
Y( 26)= 0.000000E+000
X( 27)= 4.000000E+000
Y( 27)= 0.000000E+000
X( 28)= 4.250000E+000
Y( 28)= 0.000000E+000
X( 29)= 4.500000E+000
Y( 29)= 0.000000E+000
X( 30)= 4.800000E+000
Y( 30)= 0.000000E+000
X( 31)= 5.000000E+000
Y( 31)= 0.000000E+000
X( 32)= 5.200000E+000
Y( 32)= 0.000000E+000
X( 33)= 5.400000E+000
Y( 33)= 0.000000E+000
X( 34)= 5.800000E+000
Y( 34)= 0.000000E+000
X( 35)= 5.850000E+000
Y( 35)= 0.000000E+000
X( 36)= 5.900000E+000
Y( 36)= 0.000000E+000
X( 37)= 6.000000E+000
Y( 37)= 1.500000E+000

A COEFFICIENTS     B COEFFICIENTS

 5.000000E-001
 8.262358E-001      4.768298E-001
 4.119709E-001      7.131732E-001
 2.186667E-011      6.307360E-001
-2.036138E-001      3.520154E-001
-1.614057E-001      9.246861E-002
 1.043333E-011     -7.230697E-004
 1.122901E-001      6.417231E-002
 9.651357E-002      1.666468E-001
-7.666667E-013      1.936027E-001
-7.365751E-002      1.275692E-001
```

Fast Fourier Transform

This program calculates a fast Fourier transform from a set of time domain points to a set of frequency domain points.

At the user's option, the inverse fast Fourier transform, calculating the set of time domain points from a set of frequency domain points, may also be calculated.

The method used is a modification of the basic FFT algorithm. The modified algorithm takes advantage of the fact that series data will be real and the space normally reserved for the imaginary part of the complex sequence can be used to calculate a double-length real transform. This is represented for two "N" length transforms as:

$$Z(n) = X(n) + iY(n) \qquad 0 \leqslant n < N \text{ data points}$$

The transform is:

$$Z(m) = X(m) + iY(m)$$

where

$$X(m) = \frac{Z(m) + Z(N - m)^*}{2}$$

$$Y(m) = \frac{Z(m) - Z(N - m)^*}{2i}$$

Z^* is the complex conjugate of Z.

The time series $F(n)$ is given by:

$$F(n) = X(2n) + Y(2n + 1)$$

The transformation of this is:

$$F(m) = \sum_{n=0}^{N-1} X(2n)w^{mn} + \sum_{n=0}^{N-1} Y(2n + 1)w^{mn}$$

$$= \sum_{p=0}^{N-1} X(p)w^{2mp} + \sum_{p=0}^{N-1} Y(p)w^{2mp}w^{m}$$

and:

$$F(m) = X(m) + w^{m} Y(m) \qquad (1)$$

$$F(N - m) = X^{*}(m) - \left[w^{m} Y(m)\right]^{*} \qquad (2)$$

Similarly, the inverse transform may be obtained from (1) and (2):

$$Z(m) = \frac{F(m) + F(N - m)^{*}}{2} + iw^{-m}\frac{F(m) - F(N - m)^{*}}{2}$$

$$Z(N - m) = \left[\frac{F(m) + F(N - m)^{*}}{2}\right]^{*} - iw^{-m}\left[\frac{F(m) - F(N - m)^{*}}{2}\right]^{*}$$

This is simply an interchange of $Z(m)$ and $F(m)$ in (1) and (2), and substitution of $(-w^{-m})$ for w^{m}.

Note:

1. Since $F(0)$ and $F(N)$ are real only, $F(N)$ can be stored in the imaginary location of $F(0)$, i.e., $F(1)$.
2. $w^{m} = c^{-2\pi im/2N}$. This is half the minimum value of rotation normally used in an N-point transform.
3. $*$ = complex conjugate.

The advantages gained from this adaptation of the general FFT algorithm for time series data are:

(a) A transform of twice the length can be handled with no increase in storage for input data.

(b) Since the calculation of the transform is treated as an interactive process, intermediate and final results are stored in the same locations used for input.

REFERENCES:

1. Brigham, E. O., *The Fast Fourier Transform,* Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1974. Chapter 10.
2. Cooley, J. W., and Tukey, J. W., "An Algorithm for Machine Calculation of Complex Fourier Series", Math Computation, Vol. 19, pp. 297-301, April 1965.

User Instructions

1. Insert the Math Pac cartridge into the tape transport.
2. To load the program:
   a. Type: (REW LOAD) "FFT"
   b. Press: (END LINE)
3. To start the program:
   a. Press: (RUN)
4. When TIME OR FREQUENCY DATA (T/F)? is displayed:
   a. Enter: T, if time data is to be entered.
   b. Press: (END LINE)
   c. Go to step 5.

   OR:

   a. Enter: F, if frequency data is to be entered.
   b. Press: (END LINE)
   c. Go to step 12.
5. When # OF DATA POINTS (POWER OF 2)? is displayed:
   a. Enter: The number of points (in the time domain) to be used in the calculations. This number must be a power of 2 between 4 and 512, (i.e., 4, 8, 16, 32, etc.)
   b. Press: (END LINE)
6. At this point, a check is made on the number of data points. If an error is detected, the following message will be printed:

   # OF POINTS OUT OF RANGE
   N=

   When ENTER NUMBER OF POINTS AGAIN is displayed:
   a. Enter: The number of time domain points.
   b. Press: (END LINE)
7. When TIME DOMAIN DATA: is printed and DATA POINT (J) (for J = 1 to # of time domain points) is displayed:
   a. Enter: The appropriate time domain point.
   b. Press: (END LINE)
   c. Go to step 7 if there are more points to be entered.
8. When CHANGES (Y/N)? is displayed:
   a. Enter: Y, if changes are to be made in the data.
   b. Press: (END LINE)
   c. Go to step 9.

   OR:

   a. Enter: N, if no changes are desired in the data.
   b. Press: (END LINE)
   c. Go to step 11.
9. When DATA POINT #? is displayed:
   a. Enter: The number of the data point to be changed.
   b. Press: (END LINE)
10. When DATA POINT (I)? (where I is the coordinate of the point to be changed) is displayed:
    a. Enter: The correct data point value.
    b. Press: (END LINE)
    c. Go to step 8.
11. The program will print the DC TERM, the MAX. FREQUENCY and the real and imaginary components of the resulting frequency data. For a new case, press (RUN) and return to step 4.
12. When # OF COEFFICIENT PAIRS? is displayed:
    a. Enter: The number of coefficient pairs. This number must be 1, 3, 7, 15, 31, 63, 127, or 255.
    b. Press: (END LINE)
13. At this point, a check is made on the number of data points. If an error is detected, the following message will be printed:

    # OF POINTS OUT OF RANGE
    N=

    When ENTER NUMBER OF POINTS

AGAIN is displayed:
   a. Enter: The number of coefficient pairs.
   b. Press: END LINE
14. When DC TERM? is displayed:
   a. Enter: The DC term.
   b. Press: END LINE
15. When MAX FREQUENCY TERM? is displayed:
   a. Enter: The maximum frequency term.
   b. Press: END LINE
16. When FREQUENCY DOMAIN DATA: is printed and REAL (J) (for J = 1 to # of points) is displayed:
   a. Enter: The appropriate real coefficient.
   b. Press: END LINE
17. When IMAG (J) (for J = 1 to # of points) is displayed:
   a. Enter: The appropriate imaginary coefficient.
   b. Press: END LINE
   c. Go to step 16 if there are more points to be entered.
18. When CHANGES (Y/N)? is displayed:
   a. Enter: Y, if changes are to be made in the data.
   b. Press: END LINE
   c. Go to step 19.

   OR:
   a. Enter: N, if no changes are desired in the data.
   b. Press: END LINE
   c. Go to step 22.
19. When DATA POINT #? is displayed:
   a. Enter: The number of the data point to be changed.
   b. Press: END LINE
20. When REAL (J)? (where J is the coordinate of the real point to be changed) is displayed:
   a. Enter: The coefficient.
   b. Press: END LINE
21. When IMAG (J)? (where J is the coordinate of the imaginary point to be changed) is displayed:
   a. Enter: The coefficient.
   b. Press: END LINE
22. The program will print the time domain data points obtained from the inverse fast Fourier transform. For a new case, press RUN and return to step 4.

Example 1:

```
# OF DATA POINTS= 16
TIME DOMAIN DATA:

   POINT(  1)= 0.000000E+000
   POINT(  2)= 3.826834E-001
   POINT(  3)= 7.071068E-001
   POINT(  4)= 9.238795E-001
   POINT(  5)= 1.000000E+000
   POINT(  6)= 9.238795E-001
   POINT(  7)= 7.071068E-001
   POINT(  8)= 3.826834E-001
   POINT(  9)= 0.000000E+000
   POINT( 10)=-3.826834E-001
   POINT( 11)=-7.071068E-001
   POINT( 12)=-9.238795E-001
   POINT( 13)=-1.000000E+000
   POINT( 14)=-9.238795E-001
   POINT( 15)=-7.071068E-001
   POINT( 16)=-3.826834E-001

DC TERM= 0
MAX FREQUENCY= 0

FREQUENCY DOMAIN:

          REAL              IMAGINARY
   1
    -6.250000E-013  -1.000000E+000

   2
     0.000000E+000   0.000000E+000

   3
     3.790041E-013   2.078193E-009

   4
     0.000000E+000   0.000000E+000

   5
     8.709959E-013   1.538194E-008

   6
     0.000000E+000   0.000000E+000

   7
    -6.250000E-013   2.786313E-008
```

Example 2:

```
# OF DATA POINTS= 16
TIME DOMAIN DATA:

   POINT(  1)= 1.000000E+000
   POINT(  2)= 1.306600E+000
   POINT(  3)= 1.414200E+000
   POINT(  4)= 1.306600E+000
   POINT(  5)= 1.000000E+000
   POINT(  6)= 5.412000E-001
   POINT(  7)= 0.000000E+000
   POINT(  8)=-5.412000E-001
   POINT(  9)=-1.000000E+000
   POINT( 10)=-1.306600E+000
   POINT( 11)=-1.414200E+000
   POINT( 12)=-1.306600E+000
   POINT( 13)=-1.000000E+000
   POINT( 14)=-5.412000E-001
   POINT( 15)= 0.000000E+000
   POINT( 16)= 5.412000E-001

DC TERM= 0
MAX FREQUENCY= 0

FREQUENCY DOMAIN:

          REAL            IMAGINARY
  1
     1.000010E+000 -1.000010E+000

  2
     0.000000E+000  0.000000E+000

  3
    -1.339454E-006 -1.339453E-006

  4
     0.000000E+000  0.000000E+000

  5
     6.134477E-006 -6.134475E-006

  6
     0.000000E+000  0.000000E+000

  7
    -1.502234E-005 -1.502234E-005
```

Example 3:

```
# OF COEFFICIENT PAIRS= 7

DC TERM= 0
MAX FREQUENCY TERM= 0

FREQUENCY DOMAIN DATA:

   REAL(  1)= 1.000000E+000
   IMAG(  1)=-1.000000E+000
   REAL(  2)= 0.000000E+000
   IMAG(  2)= 0.000000E+000
   REAL(  3)=-1.339500E-006
   IMAG(  3)=-1.339500E-006
   REAL(  4)= 0.000000E+000
   IMAG(  4)= 0.000000E+000
   REAL(  5)= 6.134500E-006
   IMAG(  5)=-6.134500E-006
   REAL(  6)= 0.000000E+000
   IMAG(  6)= 0.000000E+000
   REAL(  7)=-1.502200E-005
   IMAG(  7)=-1.502200E-005

TIME DOMAIN:

   DATA POINT(  1)= 9.999898E-001
   DATA POINT(  2)= 1.306587E+000
   DATA POINT(  3)= 1.414186E+000
   DATA POINT(  4)= 1.306587E+000
   DATA POINT(  5)= 9.999898E-001
   DATA POINT(  6)= 5.411945E-001
   DATA POINT(  7)= 4.000000E-012
   DATA POINT(  8)=-5.411945E-001
   DATA POINT(  9)=-9.999898E-001
   DATA POINT( 10)=-1.306587E+000
   DATA POINT( 11)=-1.414186E+000
   DATA POINT( 12)=-1.306587E+000
   DATA POINT( 13)=-9.999898E-001
   DATA POINT( 14)=-5.411945E-001
   DATA POINT( 15)=-4.000000E-012
   DATA POINT( 16)= 5.411945E-001
```

Notes

Hyperbolics

This program computes hyperbolic functions and their inverses.

Equations:

Hyperbolic Functions

$$\sinh x = \frac{(e^{x} - e^{-x})}{2}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$

$$\tanh x = 1 - \frac{2}{1 + e^{2x}}$$

Inverse Hyperbolic Functions

$$\sinh^{-1} x = \ln \left[x + (x^2 + 1)^{1/2}\right]$$

$$\cosh^{-1} x = \ln \left[x + (x^2 - 1)^{1/2}\right] \qquad x \geqslant 1$$

$$\tanh^{-1} x = ½ \ln \left[\frac{1 + x}{1 - x}\right] \qquad x^2 < 1$$

User Instructions

1. Insert the Math Pac cartridge into the tape transport.
2. To load the program:
   a. Type: REW LOAD "HYPER"
   b. Press: END LINE
3. To run the program:
   a. Press: RUN
4. When SELECT OPTION is displayed:
   a. Press: KEY #1 (SINH) to compute sinh x.
   b. Go to step 5.
   OR:
   a. Press: KEY #2 (COSH) to compute cosh x.
   b. Go to step 5.
   OR:
   a. Press: KEY #3 (TANH) to compute tanh x.
   b. Go to step 5.
   OR:
   a. Press: KEY #5 (ASINH) to compute $\sinh^{-1}$ x.
   b. Go to step 5.
   OR:

a. Press: KEY #6 (ACOSH) to compute $\cosh^{-1} x$.

b. Go to step 5.

OR:

a. Press: KEY #7 (ATANH) to compute $\tanh^{-1} x$.

b. Go to step 5.

5. When X=? is displayed:
   a. Enter: The value of x.
   b. Press: END LINE

6. After the value of the hyperbolic function is printed, go to step 4.

Examples:

```
SINH( 2.5 )= 6.05020448105
COSH( 3.2 )= 12.2866462006
TANH( 1.9 )= .956237458128
ASINH( 2.4 )= 1.60943791243
ACOSH( 90 )= 5.19292598526
ATANH(-.65 )=-.775298706205
```

Complex Operations

This program is used for calculations involving complex numbers. Three operations of complex arithmetic ($+$, $\times$, $\div$) are provided, as well as common functions of a complex variable z (z^N, $z^{1/N}$, e^z and LOGz) and the evaluation of a complex polynomial at a complex point.

Equations:

Let

$$z_k = a_k + ib_k = r_k e^{i\theta_k}$$

$$z_1 + z_2 + \ldots + z_N = (a_1 + a_2 + \ldots + a_N) + i(b_1 + b_2 + \ldots + b_N)$$

$$z_1 z_2 = (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + b_1 a_2)$$

$$z_1/z_2 = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$

$$z^N = r^N(\cos N\theta + i\sin N\theta)$$

$$z^{1/N} = r^{1/N}\left[\cos\left(\frac{\theta + 2k\pi}{N}\right) + i\sin\left(\frac{\theta + 2k\pi}{N}\right)\right], k = 0, 1, \ldots, N - 1$$

$e^Z = e^a(\cos b + i\sin b)$, where $z = a + ib$ and b is in radians

LOG(z) = LOG $r + i\theta$, where $z = re^{i\theta} \neq 0$ and $-\pi < \theta \leq \pi$

Complex polynomial:

$$(a_N + ib_N)z^N + \ldots + (a_1 + ib_1)z^1 + (a_0 + ib_0)z^0$$

User Instructions

1. Insert the Math Pac cartridge into the tape transport.
2. To load the program:
   a. Type: (REW LOAD) "COMPLX"
   b. Press: (END LINE)
3. To start the program:
   a. Press: (RUN)
4. When the keys are labelled and SELECT OPTION is displayed:
   a. Press: KEY #1 (C+) for complex addition.
   b. Go to step 5.

   OR:

   a. Press: KEY #2 (C×) for complex multiplication.

b. Go to step 7.

OR:

a. Press: KEY #3 (C/) for complex division.

b. Go to step 7.

OR:

a. Press: KEY #4 (e^Z) to raise e to a complex power.

b. Go to step 8.

OR:

a. Press: KEY #5 (Z^N) to raise a complex number to an integer power.

b. Go to step 9.

OR:

a. Press: KEY #6 (Z^1/N) to find the N^th root of a complex number. All N roots will be found.

b. Go to step 9.

OR:

a. Press: KEY #7 (POLYZ) to evaluate a complex polynomial at a complex point.

b. Go to step 11.

OR:

a. Press: KEY #8 (LOGZ) to find the natural logarithm of a complex number.

b. Go to step 8.

5. When # OF COMPLEX NUMBERS? is displayed:
   a. Enter: The number of complex numbers to be added.
   b. Press: END LINE
6. When INPUT COMPLEX NO. (REAL, IMAG) is displayed:
   a. Enter: The real and imaginary parts of the complex number, separated by a comma.
   b. Press: END LINE
   c. Repeat step 6 until all numbers have been input.
   d. Go to step 15.
7. When INPUT COMPLEX NO. (REAL, IMAG) is displayed:
   a. Enter: The real and imaginary parts of the complex number, separated by a comma.
   b. Press: END LINE
   c. Repeat step 7 to input second complex number.
   d. Go to step 15.
8. When INPUT COMPLEX NO. (REAL, IMAG) is displayed:
   a. Enter: The real and imaginary parts of the complex number, separated by a comma.
   b. Press: END LINE
   c. Go to step 15.
9. When INPUT COMPLEX NO. (REAL, IMAG) is displayed:
   a. Enter: The real and imaginary parts of the complex number, separated by a comma.
   b. Press: END LINE
10. When N=? is displayed:
   a. Enter: The value of N.
   b. Press: END LINE
   c. Go to step 15.
11. When DEGREE OF POLYNOMIAL? is displayed:
   a. Enter: The degree of the polynomial.
   b. Press: END LINE
12. When COEFF (J) (REAL, IMAG)? (for J = N to 0) is displayed:
   a. Enter: The real and imaginary parts of the coefficient, separated by a comma.
   b. Press: END LINE
   c. Repeat step 12 until all coefficients are entered.
13. When POINT (REAL, IMAG)? is displayed:
   a. Enter: The real and imaginary parts of the point to be evaluated, separated by a comma.
   b. Press: END LINE
14. The result will be printed, and when the display shows ANOTHER POINT (Y/N)? :

a. Enter: Y, (yes) to evaluate the polynomial at another point.
b. Press: END LINE and go to step 13.

OR:

a. Enter: N, (no) to display key labels.
b. Press: END LINE

15. The result of the calculation will be printed using the format (Real, Imaginary) and you are returned to step 4 for a new problem.

Example 1:

Find the 3 cube roots of 8.

```
RESULT=
( 2.000000E+000, 0.000000E+000)
RESULT=
(-1.000000E+000, 1.732051E+000)
RESULT=
(-1.000000E+000,-1.732051E+000)
```

Example 2:

Evaluate the following polynomial at the complex points (1 − i) and (3 − 5i).

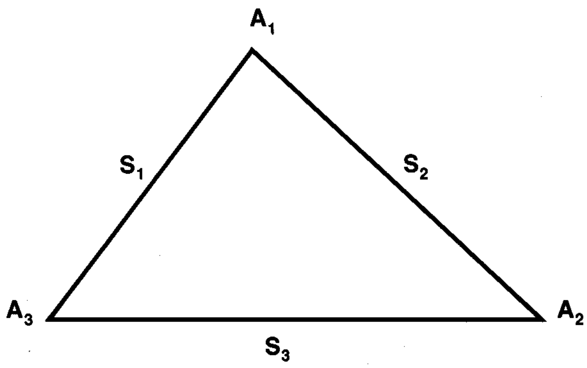
$f(z) = (3 + i)z^3 - (6 + 4i)z + (3 + 7i)$

```
RESULT=
(-1.100000E+001, 1.000000E+000)
RESULT=
(-6.190000E+002,-2.030000E+002)
```

Notes

Triangle Solutions

This program finds the dimensions of the sides and angles of a triangle, and also determines the area.



Equations:

S_1,S_2,S_3 (all sides known)

$$A_3 = 2\cos^{-1}\sqrt{\frac{P(P - S_2)}{S_1 S_3}}$$

where $P = (S_1 + S_2 + S_3)/2$

$$A_2 = 2\cos^{-1}\sqrt{\frac{P(P - S_1)}{S_2 S_3}}$$

$$A_1 = \cos^{-1}(-\cos(A_3 + A_2))$$

A_1,S_1,A_3 (two angles and included side known)

$$A_2 = \cos^{-1}(-\cos(A_3 + A_1))$$

$$S_2 = S_1 \frac{\sin A_3}{\sin A_2}$$

$$S_3 = S_1 \cos A_3 + S_2 \cos A_2$$

S_1,A_1,A_2 (side and following two angles known)

$$A_3 = \cos^{-1}(-\cos(A_1 + A_2))$$

Problem has been reduced to the A_3,S_1,A_1 configuration.

S_1,A_1,S_2 (two sides and included angle known)

$$S_3 = \sqrt{S_1^2 + S_2^2 - 2S_1S_2 \cos A_1}$$

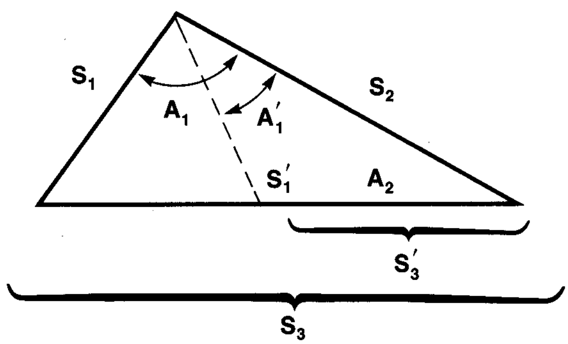
Problem has been reduced to the S_1,S_2,S_3 configuration.

S_1,S_2,A_2 (two sides and adjacent angle known)

$$A_3 = \sin^{-1}\left[\frac{S_2}{S_1} \sin A_2\right]^*$$

$$A_1 = \cos^{-1}\left[-\cos(A_2 + A_3)\right]$$

Problem has been reduced to the A_3,S_1,A_1 configuration.



Area = 1/2 S_1 S_3 sin A_3

* Note that two possible solutions exist if S_2 is greater than S_1 and A_3 does not equal 90°. Both possible answer sets are calculated.

Remarks:

- Note that the triangle described by the program does not conform to the standard triangle notation, i.e., A_1 is not opposite S_1.
- When in DEG mode, angles must be entered in decimals.
- Accuracy of solutions may degenerate for triangles containing extremely small angles.

User Instructions

1. Insert the Math Pac cartridge into the tape transport.
2. To load the program:
   a. Type: [REW LOAD] "TRANGL"
   b. Press: [END LINE]
3. To start the program:
   a. Press: [RUN]
4. When SELECT DEG OR RAD MODE (D/R)? is displayed:
   a. Enter: D, if degrees mode is desired.
   b. Press: [END LINE]
   OR:
   a. Enter: R, if radians mode is desired.
   b. Press: [END LINE]
5. When the keys are labelled and SELECT OPTION is displayed:
   a. Press: KEY #1 (SSS), if all sides of the triangle are known.
   b. Go to step 6.
   OR:
   a. Press: KEY #2 (ASA), if two angles and the included side are known.
   b. Go to step 9.
   OR:
   a. Press: KEY #3 (SAA), if one side and the following two angles are known.
   b. Go to step 12.
   OR:
   a. Press: KEY #4 (SAS), if two sides and the included angle are known.
   b. Go to step 15.
   OR:
   a. Press: KEY #5 (SSA), if two sides and the adjacent angle are known.
   b. Go to step 18.
6. When SIDE 1? is displayed:
   a. Enter: Side 1.
   b. Press: [END LINE]
7. When SIDE 2? is displayed:
   a. Enter: Side 2.
   b. Press: [END LINE]
8. When SIDE 3? is displayed:
   a. Enter: Side 3.
   b. Press: [END LINE]
   c. Go to step 21.
9. When ANGLE 1? is displayed:
   a. Enter: Angle 1.
   b. Press: [END LINE]
10. When SIDE 1? is displayed:
    a. Enter: Side 1.
    b. Press: [END LINE]
11. When ANGLE 3? is displayed:
    a. Enter: Angle 3.
    b. Press: [END LINE]
    c. Go to step 21.
12. When SIDE 1? is displayed:
    a. Enter: Side 1.
    b. Press: [END LINE]
13. When ANGLE 1? is displayed:
    a. Enter: Angle 1.
    b. Press: [END LINE]
14. When ANGLE 2? is displayed:
    a. Enter: Angle 2.
    b. Press: [END LINE]
    c. Go to step 21.
15. When SIDE 1? is displayed:
    a. Enter: Side 1.
    b. Press: [END LINE]
16. When ANGLE 1? is displayed:
    a. Enter: Angle 1.
    b. Press: [END LINE]
17. When SIDE 2? is displayed:
    a. Enter: Side 2.
    b. Press: [END LINE]
    c. Go to step 21.
18. When SIDE 1? is displayed:

a. Enter: Side 1.
b. Press: (END LINE)

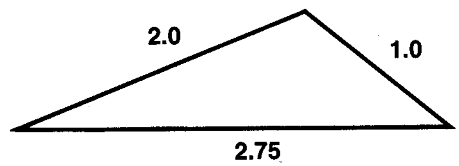
19. When `SIDE 2?` is displayed:
    a. Enter: Side 2.
    b. Press: (END LINE)

20. When `ANGLE 2?` is displayed:
    a. Enter: Angle 2.
    b. Press: (END LINE)

21. The values of the sides are printed, along with the area of the triangle, and you are returned to step 5 for a new problem.

Example 1:

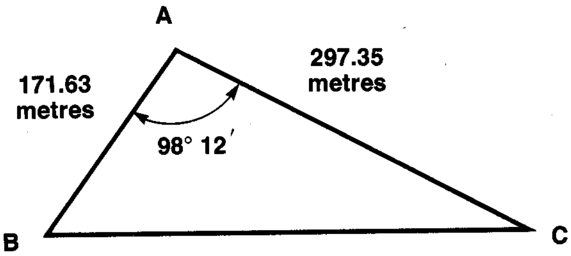
Find the angles (in degrees) and the area for the following triangle.




```
SIDE 1= 2
ANGLE 1= 129.838439978
SIDE 2= 1
ANGLE 2= 33.947926527
SIDE 3= 2.75
ANGLE 3= 16.2136334955
AREA= .767853898435
```

Example 2:

A surveyor is to find the area and dimensions of a triangular land parcel. From point A, the distances to B and C are measured with an electronic distance meter. The angle between AB and AC is also measured. Find the area and other dimensions of the triangle.



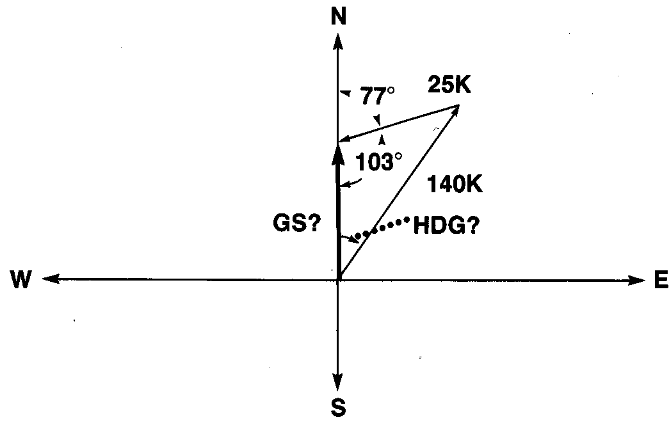
This is a side-angle-side problem where:

$$S_1 = 171.63,\ A_1 = 98.2° \text{ and } S_2 = 297.35.$$

```
SIDE 1= 171.63
ANGLE 1= 98.2
SIDE 2= 297.35
ANGLE 2= 27.8270047024
SIDE 3= 363.911776252
ANGLE 3= 53.9729952982
AREA= 25256.2094113
```

Example 3:

A pilot wishes to fly due north. The wind is reported as 25 knots at 77°. Because winds are reported opposite to the direction they blow, this is interpreted as 77 + 180 or 257°. The true airspeed of the aircraft is 140 knots. What heading (HDG) should be flown? What is the ground speed (GS)?



By subtracting the wind direction from 180 (yielding an angle of 103°), the problem reduces to a S_1, S_2, A_2 triangle.

```
SIDE 1= 140
ANGLE 1= 66.979841475
SIDE 2= 25
ANGLE 2= 103
SIDE 3= 132.240747216
ANGLE 3= 10.0201585254
AREA= 1610.64281788
```

Appendix A

The documentation for the programs contained in the Math Pac is available by executing the comments program, MTCOM. By executing this program, you can obtain the comments for any or all of the programs in the Pac. The definitions of the major variables used in each program are also given.

User Instructions

1. Insert the Math Pac cartridge into the tape transport.
2. To load the program:
   a. Type: (REW LOAD) "MTCOM"
   b. Press: (END LINE)
3. To start the program:
   a. Press: (RUN)
4. When the keys are labelled and SELECT PROGRAM is displayed:
   a. Press: KEY #1 (SIMEQ) to obtain the comments for the program SIMEQ.

   OR

   a. Press: KEY #2 (ROOTS) to obtain the comments for the program ROOTS.

   OR

   a. Press: KEY #3 (INTEG) to obtain the comments for the program INTEG.

   OR

   a. Press: KEY #4 (INTUN) to obtain the comments for the program INTUN.

   OR

   a. Press: KEY #5 (DIFEQ) to obtain the comments for the program DIFEQ.

   OR

   a. Press: KEY #6 (CHEBY) to obtain the comments for the program CHEBY.

   OR

   a. Press: KEY #7 (DONE).
   b. The program will stop.

   OR

   a. Press: KEY #8 (TOGGLE) to label the keys with the names of other programs in the Pac.
   b. Go to step 5.
5. When the keys are labelled and SELECT PROGRAM is displayed:
   a. Press: KEY #1 (FOUREQ) to obtain the comments for the program FOUREQ.

   OR

   a. Press: KEY #2 (FOURUN) to obtain the comments for the program FOURUN.

   OR

   a. Press: KEY #3 (FFT) to obtain the comments for the program FFT.

   OR

   a. Press: KEY #4 (COMPLX) to obtain the comments for the program COMPLX.

   OR

   a. Press: KEY #5 (HYPER) to obtain the comments for the program HYPER.

   OR

   a. Press: KEY #6 (TRANGL) to obtain the comments for the program TRANGL.

   OR

   a. Press: KEY #7 (DONE).
   b. The program will stop.

   OR

   a. Press: KEY #8 (TOGGLE) to label the keys with the names of other programs in the Pac.

Appendix B

Using Your Math Pac With an Eighty-Column Display

Note: The actual error messages, prompt messages, and report formats may be different than those listed in this manual due to the larger display on your computer.

Data Storage and Retrieval

On the eighty-column display Math Pac data storage and retrieval are available for SIMEQ, INTEG, INTUN, CHEBY, FOUREQ, FOURUN, and FFT programs. When running these programs, follow the steps below.

User Instructions

1. Load the selected program and press (RUN).
2. When the keys are labeled and SELECT OPTION is displayed:
   a. Press: KEY #1 (ENTER) to enter data from the keyboard or a disc file, and edit the data.
   b. Go to step 3.

   OR:

   a. Press: KEY #2 (OUTPUT) to print a copy of the data on the display or external printer and/or output the data to a disc file.
   b. Go to step 9.

   Note: If the data has not been entered from the keyboard or disc file, the program will beep, display MUST ENTER DATA FIRST!, and go to step 2.

   Note: If the program displays INPUT DATA NOT AVAILABLE, the original data values were destroyed while computing the selected program's solution. To continue, a new set of data must be entered from the keyboard or a disc file.

   OR:

   a. Press: KEY #3 (SOLVE) to execute the program's solution as described by the appropriate section in the front of this manual.
   b. After the solution has been computed, go to step 2.

   Note: If the data has not been entered from the keyboard or disc file, the program will beep, display MUST ENTER DATA FIRST!, and go to step 2.

Note: If the program displays INPUT DATA NOT AVAILABLE, the original data values were destroyed while computing the program's solution and cannot be used again. The user must enter a new set of data values from the keyboard or a disc file to continue.

OR:

a. Press: KEY #4 (HELP) for an explanation of the key functions.

OR:

a. Press: KEY #5 (DONE) to stop program execution.

3. When PRINT DATA ON INPUT: (Y/N)? is displayed:
   a. Enter: N to have the data entered from the keyboard or disc file without receiving a listing of it.
   b. Press: (END LINE)
   c. Go to step 5.

   OR:

   a. Enter: Y to have the data displayed on the screen or printed on the external printer as it is entered from the keyboard or disc file.
   b. Press: (END LINE).

4. When PLEASE SPECIFY THE PRINTER... is displayed:
   a. Enter: 1 to have the data displayed on the screen.
   b. Press: (END LINE).

   OR:

   a. Enter: the HP-IB address code of the external printer to have the data list printed on the printer.
   b. Press: (END LINE).

   Note: If the address entered is illegal, the program will beep, display INCORRECT PRINTER SPECIFICATION, and go to step 4.

5. The program will then ask ENTER THE DATA FROM KEYBOARD OR DISC: (K/D)?
   a. Enter: K if the data is to be typed in from the keyboard.
   b. Press: (END LINE).

   Note: The method of entering data from the keyboard will vary depending upon which program is being run. The user should follow the steps for entering data as listed in the appropriate section in the front of this manual.

   c. Go to step 8.

   OR:

   a. Enter: D if the data is in a disc file.
   b. Press: (END LINE).

6. When ENTER FILE NAME? is displayed:
   a. Enter: the name of the disc file containing the data array.
   b. Press: (END LINE).

7. After ENTER THE .volume label OR :msus OF THE DISC appears:
   a. Enter: the .volume label of the disc with the data file on it.
   b. Press: (END LINE).

   OR:

   a. Enter: the :msus of the disc drive that holds the disc with the data file.
   b. Press: (END LINE).

   Note: All volume labels must be preceded by a period, and all msus designators must begin with a colon. If they do not or if the file does not exist, the program will go to step 6.

8. After the data is entered, the program will go to step 2.

9. When PRINT DATA: (Y/N)? is displayed:
   a. Enter: N if you do not want a listing of the data.

b. Press: END LINE.

c. Go to step 11.

OR:

a. Enter: Y if you want a listing of the data.

b. Press: END LINE.

10. Select the printer when PLEASE SPECIFY THE PRINTER... is displayed:

a. Enter: 1 to have the data displayed on the screen.

b. Press: END LINE.

OR:

a. Enter: the HP-IB address code of the external printer to have the data list printed on the printer.

b. Press: END LINE.

Note: If the address entered is illegal, the program will beep, display INCORRECT PRINTER SPECIFICATION, and go to step 10.

c. The data will then be listed on the specified device.

11. When STORE DATA: (Y/N)? is displayed:

a. Enter: N if you do not want the data stored.

b. Press: END LINE.

c. Go to step 2.

OR:

a. Enter: Y if you want the data stored in disc file.

b. Press: END LINE.

12. When ENTER FILE NAME? is displayed:

a. Enter: the name of the file that will contain the data.

b. Press: END LINE.

13. After ENTER THE .volume label OR :msus OF THE DISC appears:

a. Enter: the .volume label of the disc to receive the data file.

b. Press: END LINE.

OR:

a. Enter: the :msus of the disc drive that holds the disc to receive the data file.

b. Press: END LINE.

Note: All volume labels must be preceded by a period and all msus designators must begin with a colon. If the volume label or msus is illegal, the program will go to step 12.

14. After the data file has been stored, the program will go to step 2.

Notes

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